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# CuTe Manual

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**Abstract**

CuTe is a C++ program to calculate the differential cross-section for electro-weak gauge boson and Higgs boson production at small transverse momentum $q_T$. It is based on resummed expressions obtained in [1, 2] using soft-collinear effective theory. The code implements NNLL resummation as well as matching to fixed-order results and uses the LHAPDF interface [3] to provide different PDF sets.

# Contents

# 1 Introduction

CuTe is a program to compute the transverse momentum distribution of processes of the Drell-Yan (DY) type. Historically the DY process [4] denoted the inclusive production of a virtual photon by quark-antiquark annihilation in hadron collisions and the subsequent decay into a lepton pair. Its characteristic features are strongly coupled initial and color-neutral final states and so the photon case can easily be generalized to $W$- and $Z$-boson production. Also Higgs production via gluon fusion belongs to the same category of processes.

The transverse-momentum distribution of DY-like processes is one of the most basic observables at hadron colliders. It is used e.g. to extract the W-boson mass and width and is of great phenomenological relevance for Higgs-production at the LHC. Especially the regime of small transverse-momentum $q_T^2 \ll M^2$ is important, because it gives the largest contribution to the total cross-section. Here $q_T$ denotes the transverse component of the boson momentum $q$, while $M^2$ is its invariant mass $q^2$. We consider:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} \quad \text{with} \quad q^2 = M^2 \gg q_T^2$$

The hierarchy between the hard scale $M$ and the collinear scale $q_T$ leads to large logarithms which spoil the perturbativity of fixed-order calculations and need to be resummed to all orders. Our approach to resummation is based on Soft-Collinear Effective Theory (SCET) and was introduced for the DY process in [5] and used to resum large logarithms to NNLL at small transverse momentum $M^2 \gg q_T^2 \gg \Lambda_{QCD}^2$. In [1] it has been extended to $Z$-boson and $W$-boson production and improved to hold even at vanishing transverse momentum $q_T^2 < \Lambda_{QCD}^2$, using an improved resummation scheme. The extension of the formulas to Higgs-boson production via gluon-fusion was done in [2].

CuTe combines the computation of these processes in one program. It is written C++ and implements the formulas given in [1] and [2]. In addition to the resummed transverse momentum distributions at NNLL, it can calculate the matching correction to the fixed-order cross-section (see [6, 7] for DY, $W$ and $Z$ production, and [8] for the Higgs case), which becomes important at medium and large transverse momenta. The code provides a variety of options such as the computation of scale and PDF uncertainties. The parton-distribution-functions (PDF) are included via the Les Houches Accord PDF (LHAPDF [3]) interface in order to give simple access to different PDF sets. Numerical calculations are done using the GNU Scientific Library (GSL).

# 2 Installation

## 2.1 Prerequisites

The code relies on the following two libraries:

- GSL - GNU Scientific Library
- LHAPDF - the Les Houches Accord PDFs[3]

## 2.2 Basic Installation

CuTe follows the standard GNU installation procedure:

1. Download the CuTe-package from http://cute.hepforge.org/.
2. Unpack the CuTe-package in an arbitrary directory (BUILD_DIR).
3. cd into BUILD_DIR.
4. Run the configure script to create the makefiles:
   ```
   ./configure
   ```
5. Build the project:
   ```
   make
   ```
6. Install the project:
   ```
   make install
   ```

This procedure installs the CuTe binary into 'PREFIX/bin/', with PREFIX='/usr/local'.

## 2.3 Configure Options

### User Defined Installation Directory

The option −−prefix changes the PREFIX to a user defined directory.

For example the command

```
./configure --prefix='/home/dwilhelm/Desktop/test'
```

will install the CuTe binary into '/home/dwilhelm/Desktop/test/bin/'.

### User Defined Library Location

The LHAPDF/GSL installation provides a script named lhapdf-config/gsl-config, which contains information about the headers, libraries and PDFset directories. It is usually contained in the subdirectory /bin of the LHAPDF/GSL installation directory. If configure cannot find the LHAPDF/GSL libraries one can specify the location by defining the variable LHAPDF_CONFIG/GSL_CONFIG.

For example, if LHAPDF was installed in '/home/dwilhelm/test/LHAPDF', the lhapdf-config script would be in '/home/dwilhelm/test/LHAPDF/bin' and the command

```
./configure LHAPDF_CONFIG='/home/dwilhelm/test/LHAPDF/bin/lhapdf-config'
```

would lead configure to the right directories.

## 3 Usage

In general CuTe expects user written infiles, which specify the process parameters, as arguments and creates corresponding outfiles. The infiles should use the name convention NAME.*in*, so the corresponding outfiles will be written in the same directory and automatically named as NAME.*out*.

For example, the command

```
CuTe '/home/dwilhelm/test/plots/testplot.in'
```

will read the file 'testplot.in' and create a file 'testplot.out' in the directory '/home/dwilhelm/test/-plots/'.

It is possible to give an arbitrary number of infiles as arguments. CuTe will work them all in a row, creating a corresponding outfile for each infile.

Running CuTe without an argument, will create an example.in file (see page 4) in the current directory and calculate the corresponding example.out.

### 3.1 Infiles

The infiles are standard textfiles. CuTe searches them for

OPTION = ...

and interprets everything after the equal sign as the value for the OPTION, until the next option appears, while line-breaks and comments (everything after a "#" sign) are ignored. The order of the options is arbitrary.

The example.in (see page 4) can be used as template for new infiles, since it contains all possible keywords, some are just commented out.

### 3.2 Outfiles

The outfiles start with a header containing information about the CuTe version, date of calculation and a copy of the infile. The whole header is market as a comment using #.

The results are given in a tabulator separated table. The last commented line yields the label of each column. Which columns are shown depends on the options.

**Variables:** The first columns state the variables of the cross-section.

**q_T** The transverse momentum of the boson.

**y** The rapidity of the boson (only shown for the double differential cross-section $\frac{d^2\sigma}{dq_T dy}$)

**l** The non-perturbative scale $\Lambda_{NP}$ (only shown by using non-perturbative effects).

**mu** The resummation scale $\mu$.

**Final results:** Depending on the infile one of those columns is given.

**d2s/dqdy** The double differential cross-section $\frac{d^2\sigma}{dq_T dy}$

**ds/dq** The single differential cross-section $\frac{d\sigma}{dq_T}$

**Intermediate results :** If the final result is composed of different intermediate ones, each will be shown in a separate column.

**RES** The resummed result.

**RES_FO** The resummed result expanded to fixed-order.

**FO** The fixed-order result.

**Uncertainties:** After each result column, the associated uncertainties are given in the following columns.

**D PDF** The symmetric error for hessian PDF-uncertainties and the standard-deviation for other sets.

**D PDF+** The +-error for hessian PDF-uncertainties.

**D PDF-** The −-error for hessian PDF-uncertainties.

**D mu+** The +-error for scale-uncertainties.

**D mu-** The −-error for scale-uncertainties.

```
#        example infile
#        <- comments
#        All mass afflicted parameters in GeV

#——————— Process ———————
collision        =      PP                       #[PP; PPbar; PbarPbar]
production       =      H                        #[DY; H; Z; W]
M                =      125                      #[double] GeV
sqrts            =      8000                     #[double] GeV
#ckm             =      Vud Vus Vub ... Vtb      #[double ... double]
#——————— Plot ———————
q_T              =      0.1 0.2 0.3 0.4 0.6 0.8  #[double ... double] GeV
                        1. 1.5 2. 3. 4. 5. 5.5
                        6. 6.5 7. 7.5 8. 8.5 9.
                        9.5 10. 10.5 11. 11.5
                        12. 12.5 13. 13.5 14.
                        14.5 15. 16. 17. 18. 19.
                        20. 22.5 25. 27.5 30.
                        32.5 35. 40. 45. 50.
                        55. 60.
#y               =      -1 -0.5 0. 0.5 1         #[double ... double]
accuracy         =      1e-3                     #[double]
#——————— Scales ———————
#mu              =      2 3 4 5 6 7 8 9 10       #[double ... double] GeV
#q_star          =      7.7                      #[double] GeV
#mu_h            =      125                      #[double] GeV
mu_t             =      172.6                    #[double] GeV
#ScaleError      =      false                    #[true; false]
#——————— Resummation ———————
resummation      =      RES                      #[RES, RES_FO, FO, M_CORR, MATCHED]
order            =      1                        #[int]
improved         =      true                     #[true; false]
PiQuadrat        =      true                     #[true; false]
#exponent        =      false                    #[true; false]
#MScheme         =      NAIVE                    #[NAIVE; CONST; VAR]
#——————— Hadronic effects ———————
#cutoff          =      GAUSS                    #[NO; HARD; DIPOLE; GAUSS]
#LambdaNP        =      0.      0.3      0.6     #[double ... double] GeV
#——————— PDFs ———————
PDFset           =      MSTW2008nnlo68cl         #[string]
#PDFerror        =      NO                       #[NO; HESSIAN; GAUSSIAN]
```

### 3.3 Example

We now give a brief explanation of the 'example.in' shown above. The full set of options are explained in detail in the next section 3.4.

**Process:** The first options define the process to calculate, here Higgs production at the LHC, thus $PP \to H$ at a center-of-mass energy of 8000 GeV, with a Higgs mass of 125 GeV. The ckm option is commented out, because it only effects $W$-production and even there the default values should suffice.

**Plot:** To generate a plot over the full range of small transverse momenta a variety of $q_T$ values are stated. The rapidity option is commented out to obtain the single differential cross-section $\frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$, otherwise CuTe would calculate the double differential cross-section $\frac{\mathrm{d}^2\sigma}{\mathrm{d}q_T\mathrm{d}y}$ for every pair ($q_T$,$y$). The relative accuracy of the numerical integrations is set to one per mill.

**Scales:** The renormalization scale option mu and the q_star option are commented out, thus CuTe will use the default choice $\mu = q_T + q_*(M)$, where $q_*(M)$ is the numerical solution to equation (19) in [1]. This choice should give the best scaling behavior, and prevents from running into strong coupling at $q_T < \Lambda_{QCD}$. The hard scale (mu_h), associated with the hard momentum transfer, is set to the default value M. The top-quark (mu_t) will be integrated out at the top-quark mass ($\sim$ 172.6 GeV). The computation of scale uncertainties is switched off.

**Resummation:** CuTe will calculate the resummed (RES) result to order 1, which corresponds to NNLL. Using the improved resummation scheme, which includes higher order terms, yields NNLL accuracy both at small and also at very small q_T. The $\pi^2$-resummation [9] is turned on, to improve the resummation of the hard function. The perturbative-expansion-scheme exponent is turned off by default and the matching scheme does not effect the resummed calculation.

**Hadronic effects:** No hadronic effects are considered, thus the cut-off shape (cutoff) option and the cut-off scale (LambdaNP) are commented out.

**PDFs:** As PDF one of the MSTW sets is chosen and no uncertainties will be calculated. A list of all possible PDF sets can be found at http://lhapdf.hepforge.org/pdfsets.

### 3.4 Options

**collision** **PP; PPbar; PbarPbar**

Specifies the initial state, thus if the colliding hadrons are protons or anti-protons.

**production** **DY; H; Z; W**

Specifies the final state.

- **DY** Drell-Yan production, thus creation of a virtual photon, with subsequent decay into a lepton pair.
- **H** On-shell Higgs boson production.
- **W** On-shell W-boson production. CuTe calculates always the $W^+$-cross-section. One can produce the $W^-$-cross-section by charge conjugation, thus changing the collision parameter from PP to PbarPbar and vice versa.
- **Z** On-shell Z-boson production.

**M** **[double] GeV**

The mass of the produced boson in GeV.

**sqrts** **[double] GeV**

The center of mass energy of the colliding hadrons in GeV, for example 8000 GeV for the LHC in 2012.

**ckm** **[double. . . double], Default:** $V$

The CKM-matrix only effects the W-boson production. The ckm option expects 9 values in the order:

$$V_{ud}\; V_{us}\; V_{ub} V_{cd}\; V_{cs}\; V_{cb} V_{td}\; V_{ts}\; V_{tb}\; .$$

The default values are given by:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97427 & 0.22534 & 0.00351 \\ 0.22520 & 0.97344 & 0.0412 \\ 0.00867 & 0.0404 & 0.999146 \end{pmatrix} .$$

**q_T** **[double. . . double] GeV**

The transverse momentum $q_T$ of the produced boson in GeV.

**y** **[double. . . double], Default:** $\frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$

The rapidity of the produced boson. If this Keyword is set, CuTe will calculate the double differential cross-section $\frac{\mathrm{d}^2\sigma}{\mathrm{d}q_T \mathrm{d}y}$. Otherwise, it will integrate over $y$, thus calculate the single differential cross-section $\frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$.

**accuracy** **[double], Default: 1e-4**

Defines the relative numerical accuracy for integrations.

**mu** **[double. . . double] GeV, Default:** $\mu = q_T + q_*$

The renormalization scale in GeV. If mu has not been set, CuTe will use q_T + q_star as default.

**q_star** **[double] GeV, Default:** $q_*(M)$

If the mu option has not been set, CuTe will use q_T + q_star as renormalization scale, so q_star is an offset, which can ensure not to run into strong coupling at very small transverse momenta. Otherwise this Keyword has no effect.
The default value of q_star is given by the numerical solution of equation (19) in [1], which is depending on the mass M, the produced boson and the definition of the strong coupling in the chosen PDF set. This choice should give the best scaling behavior

**mu_h** **[double] GeV, Default: M**

The hard scale, associated with the hard momentum transfer, in GeV. For fixed order calculations this has no effect. The default value is the mass of the boson M.

**mu_t** **[double] GeV, Default: mu_h**

The scale where the top-quark is integrated out in GeV. This has no effect for the gauge bosons, only the Higgs production via gluon fusion is influenced. It should be chosen in the region of the top quark mass.

**ScaleError** **[true;false], Default: false**

If this keyword is true, CuTe will calculate the scale variation depending on the renormalization scale mu. Instead of calculating the cross-section only for one renormalization scale $\mu$, it will calculate it for

$$\tfrac{1}{2}\mu,\quad \tfrac{4}{6}\mu,\quad \tfrac{5}{6}\mu,\quad \mu,\quad \tfrac{4}{3}\mu,\quad \tfrac{5}{3}\mu,\quad 2\mu$$

and extract the scale uncertainties as the difference of the center value to the maximum and minimum of the other values.

**resummation** **RES, RES_FO, FO, M_CORR, MATCHED, Default: MATCHED**

**RES** The resummed result based on [1, 2].

**RES_FO** Gives the resummed result expanded to fixed-order, thus omits term of higher order in $\frac{q_T^2}{M^2}$ compared to the standard fixed-order results (FO) and the resummation effects in RES.

**FO** The standard fixed order calculation (see [6, 7] for DY, $W$ and $Z$ production, and [8] for the Higgs case).

**M_CORR** The matching correction. The resummed result (RES) omits terms of higher order in $\frac{q_T^2}{M^2}$ which become important at large $q_T$. The matching correction provides these terms. They are given by subtracting RES_FO of FO to avoid double counting of terms.

**MATCHED** The matched cross-section is given by adding M_CORR to RES.

**order** **[int]**

Specifies the perturbative order CuTe will include. For the resummed cross-section (RES) the value of 0 corresponds to NLL, 1 to NNLL. In fixed-order calculation (RES_FO, FO) the values correspond to LO and NLO. At LO the fixed-order cross-sections are always 0, since they are proportional to a dirac-delta-distribution for $q_T \equiv 0$.
This option has no effect on the order of the perturbative strong coupling value $\alpha_s$, since CuTe always uses the $\alpha_s$-function provided by the choosen PDF set.

**improved** **true;false, Default: true**

This keyword states if CuTe will use the improved resummation scheme [1], which is relevant to obtain predictions valid at very low q_T.

**PiQuadrat** **true;false, Default: true**

Since the wilson coefficients are evaluated at a time-like scale $-M^2$, the hard function receives large $\pi^2$ contributions in every order. By extending $\alpha_s(\mu^2)$ to the complex plane, an imaginary choice of $\mu_h$ resums these contributions to all orders. This keyword switch $\pi^2$-resummation [9] on.
Usually the $\alpha_s(\mu^2)$ value provided by the PDF set is used. To get the value for time-like choices ($\alpha_s(-\mu^2)$), CuTe uses its own perturbative strong coupling function, normalized to the PDF value $\alpha_s(\mu^2)$ and using the same order as the PDF.

**exponent** **true;false, Default: false**

Selects the perturbative expansion scheme. Instead of expanding the cross-section itself in $\alpha_s$, one can also expand everything in the exponent, as is natural for a resummed computation since the LL and NLL terms have to be exponetiated in any case.

**false** $\exp\{\sum_n \alpha_s^n C_n\} = \mathrm{e}^{C_0}\left[1 + \alpha_s C_1 + \alpha_s^2 \left(C_1^2 + \frac{1}{2}C_2\right)\right] + \mathcal{O}\left(\alpha_s^3\right)$

**true** $\exp\{\sum_n \alpha_s^n C_n\} = \exp\left\{C_0 + \alpha_s C_1 + \alpha_s^2 C_2\right\} + \mathcal{O}\left(\alpha_s^3\right)$

**MScheme** **NAIVE; CONST; VAR, default: NAIVE**

This keyword includes the resummation of the hard function into the matching correction, by defining the factor $H$ in the following equation.

$$\text{M\_CORR} = H \cdot (FO - RES\_FO)$$

**NAIVE** $H = 1$, thus no resummation is included for the matching correction.

**CONST** $H = H(\mu)$, $H$ is the full resummed hard function from the resummed (RES) result. In the calculation of the scale uncertainties the center value $\mu$ is always used for $H$.

**VAR** Similar to CONST, but $\mu$ is varied the same way as in the scale uncertainty calculation.

**cutoff** **NO; HARD; DIPOLE; GAUSS, Default: NO**

To include non-perturbative effects in the generalized PDFs ([1] equation (35),(36)) one can introduce a cutoff.

**NO** No cutoff.

**HARD** A hard cutoff.

**DIPOLE** A dipole-like cutoff.

**GAUSS** A gaussian cutoff

The cutoff scale is LambdaNP

**LambdaNP** **[double...double] GeV, Default: 0**

The cutoff scale in GeV (should be of the order of the proton mass, thus 0 to 1 GeV). For each value of LambdaNP a separate outfile will be created following the name convention "INFILENAME_l#.out" with "#" the serial number of LambdaNP.

**PDFset** **[string]**

Specifies the Name of the LHAPDF set. The corresponding PDF set must be installed before running CuTe. A list of all possible PDF sets can be found at http://lhapdf.hepforge.org/pdfsets.

**PDFerror** **NO; HESSIAN; GAUSSIAN, Default: NO**

There are two possibilities of calculating the PDF uncertainties. The choice depends on the used PDF set. Many PDF sets (like MSTW) consist of one center value $\phi_0$ and any number of eigenvalues in two directions $\phi_i^{\pm}$, while new PDF sets (like NNPDF [10]) often provide a statistical ensemble of values $\phi_i$. In the first case one needs to use the hessian option, in the latter the gaussian error scheme.

**HESSIAN** for PDF sets with center and Eigenvalues. $\Delta, \Delta_+$ and $\Delta_-$ like in [11]

**GAUSSIAN** for PDF sets of equally weighted values. $\Delta$ represents one standard deviation from the average.

## References


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